4493. Proposed by Nguyen Viet Hung.

Find all real numbers x,y such that

$$\left\{\frac{x+2y+1}{x^2+y^2+7}\right\} = \frac{1}{2}.$$

where $\{a\}$ denotes the fractional part of a.

Solution by Arkady Alt, San Jose ,California, USA.

Let $n := \left\lfloor \frac{x+2y+1}{x^2+y^2+7} \right\rfloor \in \mathbb{Z}$. Then $\frac{x+2y+1}{x^2+y^2+7} = n+\frac{1}{2} \Leftrightarrow$

$x^2+y^2+7 = \frac{2(x+2y+1)}{2n+1} \Leftrightarrow x^2+y^2-\frac{2x}{2n+1}-\frac{4y}{2n+1} = \frac{2}{2n+1}-7 \Leftrightarrow$

$\left(x-\frac{1}{2n+1}\right)^2+\left(y-\frac{2}{2n+1}\right)^2 = \frac{5}{(2n+1)^2}+\frac{2}{2n+1}-7 \Leftrightarrow$

$\left(x-\frac{1}{2n+1}\right)^2+\left(y-\frac{2}{2n+1}\right)^2 = -\frac{4n(7n+6)}{(2n+1)^2}$ implies $4n(7n+6) \le 0 \Leftrightarrow$

$-\frac{6}{7} \le n \le 0 \overset{n\in\mathbb{Z}}{\Leftrightarrow} n = 0$. Hence, $(x-1)^2+(y-2)^2 = 0 \Leftrightarrow (x,y) = (1,2)$.